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ANALYTICAL METHOD FOR CALCULATION OF TEMPERATURE OF THE PRODUCED WATER IN GEOTHERMAL WELLS

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Abstract

As fluids move through a wellbore, there is a transfer of heat between the fluids and the earth due to the difference between the fluids and the geothermal temperatures. This type of heat transmission is involved in drilling and all producing operations.

H. J. Ramey developed in 1962 an approach of solution which investigated the wellbore heat transmission to provide engineering methods useful in production and injection operations.

Nowadays the energy production in the case of the renewable energies particularly in the case of the geothermal energy, which has generated a considerable interest the past few years , we need not only quantitative but qualitative knowledge of wellbore heat transfer too with the invention of computer technologies.

This study, which presents a simple mathematical estimation of the temperature of produced geothermal hot water, is the refinements of the so-called Ramey-theory. The solution supposes that the heat transfer in the wellbore is steady-state, while the transfer to the earth is unsteady radial conduction.

Fields and calculated results of Hungarian production wells of Zsori-4 and Bogács (4-17) , are presented and analyzed to establish the appropriateness and the usefulness of the study.

keywords: Calculation of temperature; produced water, geothermal well, analytical approach .

1 Introduction

The geological and drilling researches proved that the geothermal gradient depends on the depth i.e. the temperature of the produced hot water varies from the bottomhole to the wellhead.
Clearly, the heat losses between the surface and the production layer can be extremely important during the process. The purpose of the present study to refine the approach of Ramey for wellbore heat transmission.
In his theory, he considered a mean geothermal gradient, a mean density of layers, a mean specific heat value, a mean thickness of layers to calculate the temperature.
On the over hand, we consider the flowing parameters, the hydrogeologic characteristics of the flowing hot water and the hydrogeologic characteristics of the layers
The following equations were developed under the assumptions that physical and thermal properties, the earth and wellbore fluids do not change with the temperature , the heat will transfer radially in the earth and the heat transmission in the wellbore is rapid, comparing to the heat flow in the geological formation, and thus, can be represented by steady-state solutions.

2 Temperature distribution and heat transfer

The internal energy equation of the flow is defined by Neumann differential equation as follows:

$$\rho \frac{dE}{dt} - F{:}S = div(K grad\, T) \qquad (1)$$

where E the internal energy, ϱ the produced hot water density, F the tension tensor of fluid flow, S the deformation tensor, K the thermal conductivity and T the temperature.
If the fluid flow is incompressible then $E = \rho c_v T$, where c_v is the specific heat at constant volume of fluid. The term (F:S) is the product of the work due to the variation of volume. The work due to the friction can be negligible. Thus we can write F:S=0.
The right hand member of the equation (1) defines the heat transfer from the fluid to the earth. If we consider K as the constant thermal conductivity and expressing the variation of the internal energy $\frac{dE}{dt}$ as follows

$$\frac{dE}{dt} = \frac{\partial E}{\partial t} + \vec{v}\, grad\, E \qquad (2)$$

The energy equation takes the form

$$\rho c_v \frac{\partial T}{\partial t} + \varrho\, c_v \vec{v}\ \ gradT = Kdiv(gradT)\ in\ \hat{V} = V \times]0,+\infty[\qquad (3)$$

which leads to

$$\rho c_v \frac{\partial T}{\partial t} + \rho c_v v \frac{\partial T}{\partial z} - K\frac{\partial^2 T}{\partial z^2} = 0\ \ in\ \hat{V} \qquad (4)$$

Where $\vec{v} = (0,0,v)$ the velocity of the flow, gradT =(0,0, $\frac{\partial T}{\partial z}$)
We adopt the following assumptions:

- the domain V and the surfaces A_1 , A_2 are defined as :

$$V = \{(x,y,z) \in \mathbb{R}^3 : x^2 + y^2 \le R_K^2;\ \ 0 \le z \le z_a\} \qquad (5)$$

$$A_1 = \{(x,y,z) \in \mathbb{R}^3 : x^2 + y^2 = R_K^2;\ \ 0 \le z \le z_a\ \} \qquad (6)$$

$$A_2 = \{(x,y,z) \in \mathbb{R}^3 : x^2 + y^2 < R_K^2;\ \ z = 0; z = z_a\ \} \qquad (7)$$

$$A = A_1\ \cup\ A_2 \qquad (8)$$

- the vertical axis of symmetry of the well coincides with the z-axis oriented down whose origin lies on the surface.
- the heat conduction is a slow transient heat conduction; thus $\frac{\partial T}{\partial t} = 0$.

Integrating the equation (3) on the domain V , we can write

$$\int_V \rho\, c_v \vec{v}\ gradT dV - \int_V Kdiv(gradT)\ dV = 0 \qquad (9)$$

which, using the Gauss-Ostrogradski surface integral theorem, leads to

$$\int_A \rho\, c_v v T dA - \int_V KgradT\ dA = 0 \qquad (10)$$

According to the continuity equation, we can write

$$\int_{\hat{A}_1} v dA = \int_{\hat{A}_2} v\ dA = \pi\, R_{1B}^2\, v \qquad (11)$$

Where $\hat{A}_1$ and $\hat{A}_2$ stand for the two cross-sections perpendicular to the domain V. We consider that the temperatures of the fluid flow in the depths z and ***z+dz*** are respectively ***T*** and ***T + dT***. Then the heat conduction $q^{(1)}$ can be written as follows:

$$q^{(1)} = \int_{\hat{A}_1} \rho\, c_v v T dA \qquad (12)$$

$$q^{(1)} = \pi\, R_{1B}^2\, \rho\, c_v v(T + dT - T) = \pi\, R_{1B}^2\, \rho\, c_v v dT \qquad (13)$$

During the heat transfer, the heat conduction out the casing wall $q^{(2)}$ must be taken into account and computed as follows:

$$q^{(2)} = \int_{\hat{A}_2} KgradT\overrightarrow{dA} = 2\pi R_{1B}\varphi(T_V - T_F)dz \qquad (14)$$

Where φ is the over-all heat transfer coefficient. According to the continuity equation, the heat loused by the fluid is equal to the heat transferred by the casing, i.e. $q^{(1)} = q^{(2)}$ which leads to

$$\pi\, R_{1B}^2\, \rho\, c_v v dT = 2\pi R_{1B}\varphi(T_V - T_F)dz \qquad (15)$$

Implying

$$(T_V - T_F) = \frac{R_{1B}\, \rho\, c_v v}{2\varphi}\frac{dT_V}{dz} \qquad (16)$$

According to Ramey, the rate of heat conduction from the casing to the surrounding formations can be expressed as follows:

$$q^{(3)} = \frac{2\pi K}{f(Fo)}(T_F - T_K)dz \qquad (17)$$

Where $f(Fo)$ is the dimensionless function depending of Fourier number, T_K is the geothermal temperature which is linear function of the depth, i.e. $T_K = T_a + m.z$ with T_a , the temperature on the lower limit the layer. In the surrounding formations the heat conduction rates $q^{(1)}$ *and* $q^{(3)}$ are equals; then we can write

$$\pi\, R_{1B}^2\, \rho\, c_v v dT_V = \frac{2\pi K}{f(Fo)}(T_F - T_K)dz \quad (18)$$

Which leads to:

$$T_F - T_K = \frac{R_{1B}^2\, \rho\, c_v v f(Fo)}{2K}\frac{dT_V}{dz} \qquad (19)$$

Summing the equations (16) and (19), we can determinate the temperature T_V of the produced hot water as follows:

$$\frac{dT_V}{dz} = \frac{2K\varphi}{\rho\, c_v v R_{1\,B}(K + \varphi.R_{1B}.f(Fo))}(T_V - T_K) \qquad (20)$$

Setting

$$\frac{2K\varphi}{\rho\, c_v v R_{1\,B}(K + \varphi.R_{1B}.f(Fo))} = \frac{1}{\Gamma}$$

we can write

$$\frac{dT_V}{dz} = \frac{1}{\Gamma}(T_V - T_K) \Leftrightarrow$$

$$\Gamma\frac{dT_V}{dz} = T_V - T_a - m.z \qquad (21)$$

With the boundary condition, z=H, $T_V = T_{Va}$; and using the method of variation of constants, the solution of the equation(21) takes the following form:

$$T_V = (T_{Va} - T_a - m(H + \Gamma)e^{\frac{z-H}{\Gamma}} + T_a + m(z + \Gamma) \qquad (22)$$

where T_{Va} is the temperature of the produced water on the lower limit of the layer.

The obtained formula allows us to calculate the temperature of the produced water varying from the bottomhole to the wellhead, knowing the temperatures of the lower limit of the layer and those of the produced water on the lower limit of the layer.

3 Determination of the global heat transfer coefficient φ

Before calculating the temperature T_V, the coefficient φ can be determinated by the following formula:

$$\frac{1}{\varphi} = \frac{1}{h} + \frac{R_{1B}}{k_{cas}}ln\frac{R_{1k}}{R_{1B}} + \frac{1}{h_{an}}\frac{R_{1B}}{R_{1K}} + \frac{R_{1B}}{k_{cas}}ln\frac{R_{2K}}{R_{2B}} + \frac{R_{1B}}{k_{cem}}ln\frac{R_F}{R_{2K}}$$

(23)

Where h is the heat transfer coefficient between the casing and the hot water. In the case of hydraulically smooth pipe we can calculate it with as follows

$$h = 0{,}0168\frac{k_{water}}{2R_{1B}}Re^{0{,}84}\,Pr^{0{,}4}$$

where Re is the Reynolds number $Re = \frac{2vR_{1B}\,\rho_w}{\eta_w}$

ρ_w the water density and η_w the water viscosity.

$Pr = \frac{C_w\eta_w}{k_w}$ is the Prandtl number.

In the case of turbulent flow, h will be calculated as follows:

$$h = 0{,}040\frac{\rho.v.R_{1B}\sqrt{\frac{f}{2}}}{\eta_w}(\frac{C_w\eta_w}{k_w})$$

f the resistance coefficient of the casing,

$$\frac{1}{\sqrt{f}} = 2log\frac{R_{1B}}{k_r} + 1{,}74$$

K_R the roughness coefficient of the casing

The $f(Fo)$ the Fourier transient function may be calculated as follows. If the Fourier number

$$Fo = \frac{k_k\; t}{\rho_k.C_k.R_F^2} \ll 1$$

When the water production begins, then

$f(Fo) = \sqrt{\pi.Fo}$

If $Fo \gg 1$, then

$$f(Fo) = \frac{ln\sqrt{Fo}}{2} - 0{,}405$$

is usable.

4 Determination of the heat transfer coefficient h_{GY} in the annulus between the casing and the layers

Between the casing and the layers there is a heat conduction , which may be expressed as follows:

$$\frac{R_{1B}}{R_{1K}}.\frac{1}{h_{GY}} = \frac{R_{1B}}{k_{SZ}}ln\frac{R_{2B}}{R_{1K}}$$

Where k_{sz} is the conduction coefficient of the insulator which is in the annulus.

If we have fluid between the tubing and the casing and neglecting the internal energy, the heat transfer coefficient may be calculated by the following formula;

$$h_{GY} = \frac{0{,}049.k_w.(Gr.\mathrm{Pr})^{1/3}.Pr^{0{,}074}}{R_{1B}ln\frac{R_{2B}}{R_{1K}}}$$

The symbol Gr is the Grashof number can be calculated as follows:

$$Gr = \frac{(R_{2B} - R_{1K})^3.g.\rho_w^2.\beta_w.(T_{1K} - T_{2B})}{\eta_w^2}$$

$$T_F = \frac{T(f(t)) + \frac{k_k}{R_{1B}.\varphi}T_{k\infty}}{f(t) + \frac{k_k}{R_{1B}.\varphi}}$$

And

$$T_{2B} = T_F - \left[\frac{ln\frac{R_F}{R_{2k}}}{k_{cem}} + \frac{ln\frac{R_{2k}}{R_{2B}}}{k_{cas}}\right].R_{1B}.\varphi.(T_F - T_f$$

(See figure 1; figure 2 and figure 3 for the different radius, temperatures used in the paper)

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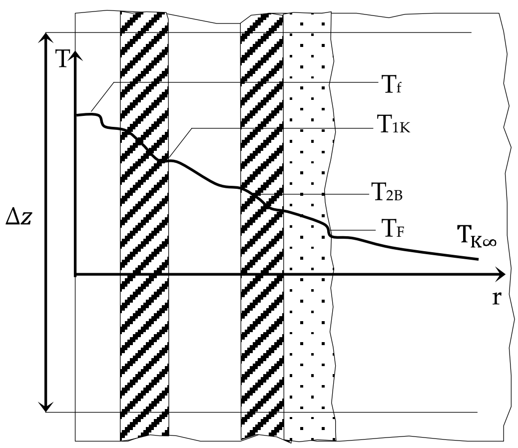


Figure 1 : Nomenclature of temperatures

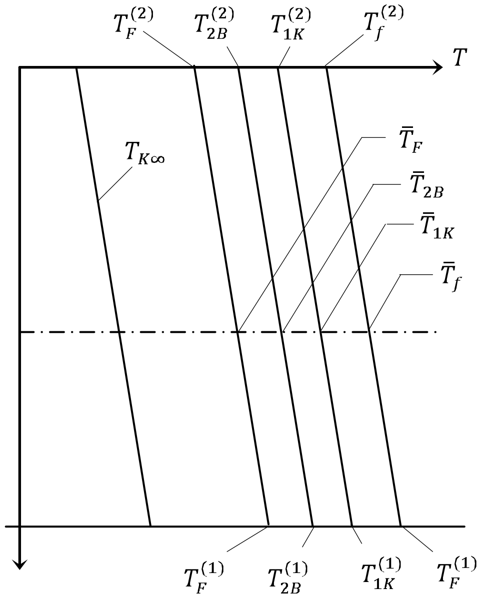


Figure 2 : Different temperatures for iterations

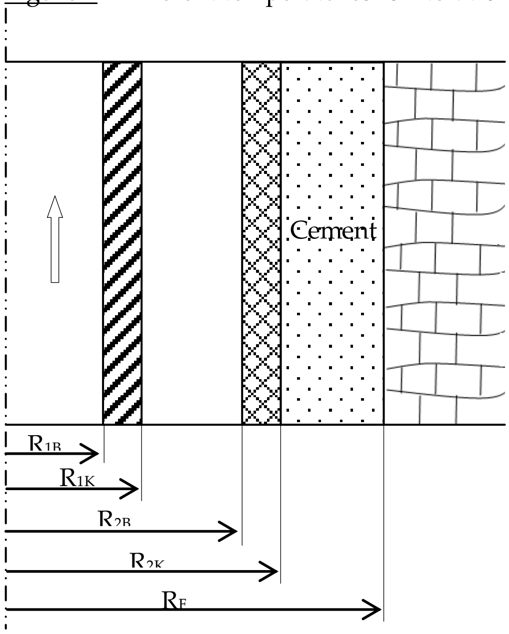


Figure 3: Different used radius

Using this method for calculation of geothermal hot water by computering the formulas above , we calculate a sample calculation of the temperature of the geothermal production wells of ZSORI and of BOGÁCS .

5 Sample calculation of the temperature of thermal hot water well Zsori-4

The thermal hot water well Zsori-4 was drilled in 1969 and was 562,5m deep. The bottomhole measured temperature was 54,5 °C and the flowing hot water measured temperature on the wellhead was 49°C. In the case of the annulus was filled of thermal water, the calculated temperature on the wellhead is 54,23°C.

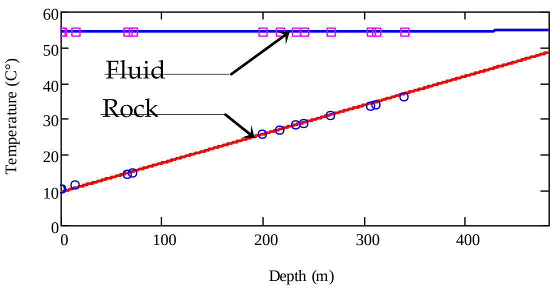
DATA OF THE WELL ZSORI-4		
Number of layers		16
Characteristics of the thermal water		
Water density (kg/m^3)		988
Water specific heat $(J/kg.K)$		4185
Water dynamic viscosity $(N.s/m^2)$		$5{,}49.10^{-4}$
Water compressibility (m^2/N)		$1{,}8.10^{-4}$
Characteristics of well		
Depth of casing (m)		547
Cement plug length in annulus (m)		39
Roughness of tubing (m)		$2{,}05.10^{-4}$
Thermal conductivity of insular $(W/m.K)$		0.0349
Thermal conductivity of steel $(W/m.K)$		52
Thermal conductivity of cement $(W/m.K)$		0.8
Mean earth temperature at wellhead $(°C)$		10
Reciprocal of geothermal gradient $(m/°C)$		12.29
Radius of hole (m)		0.3493
Radius of casing (m)		0.2245
Radius of tubing (m)	inside	0.124
	outside	0.133

CHARACTERISTICS OF LAYERS			
Thickness sheets(m)	Thermal Cond. $(W/m°K)$	Density (kg/m")	Specific heat $(J/kg.°K)$
1.5	1.25	1700	1670
13.5	0.84	1800	870
51	1.25	1800	920
5	0.84	1800	840
128	0.90	1900	860
18	1.30	1900	921
16	0.84	1800	840
7	1.30	1900	921
27	0.84	1800	850

40	1.20	1900	860
5	1.30	1900	921
28	0.90	1900	850
168	0.84	1800	840
3	0.84	1800	840
36	0.84	1800	840
1	0.90	1900	860

The results are in the following table.

Depth (m)	Temperature of layer(°C)	Temperature of Fluid (°C)
0	10	54,23
1,5	10,09	54,23
15,0	11,30	54,25
66,0	14,37	54,33
71,0	14,81	54,33
199,0	25,50	54,45
217,0	26,54	54,46
233,0	27,97	54,47
240,0	28,38	54,48
267,0	30,80	54,50
307,0	33,30	54,52
312,0	33,59	54,53
340,0	35,93	54,54
508,0	51,01	54,59
511,0	51,28	54,59
547,0	54,51	54,59
562,5	54,59	54,59



Production well of ZSORI

6 Sample calculation of the temperature of thermal hot water well of Bogács

The thermal hot water well of Bogács was drilled in 1959 and was 482m deep. The bottomhole temperature was 76,6 °C and the flowing hot water measured temperature on the wellhead was72,8 °C. In the case of the annulus was filled of thermal fluid, the calculated temperature on the wellhead is 75,90°C.

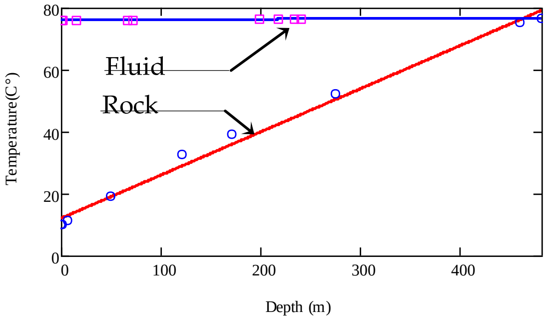
DATA OF THE WELL BOGÁCS		
Number of layers		8
Characteristics of the thermal water		
Water density (kg/m^3)		975
Water specific heat $(J/kg.K)$		4190
Water dynamic viscosity$(N.s/m^2)$		$3{,}82.10^{-4}$
Water compressibility (m^2/N)		$1{,}8.10^{-4}$
Characteristics of well		
Depth of casing(m)		172
Cement plug length in annulus(m)		26
Roughness of tubing (m)		$2{,}05.10^{-4}$
Thermal conductivity of insular $(W/m.K)$		0.0349
Thermal conductivity of steel $(W/m.K)$		52
Thermal conductivity of cement $(W/m.K)$		0.8
Mean earth temperature at wellhead $(°C)$		10
Reciprocal of geothermal gradient $(m/°C)$		7.231
Radius of hole top (m)		0.17
Radius of hole bottom (m)		0.12
Radius of casing inside (m)		0.124
Radius of casing outside (m)		0.133
Radius of tubing (m)	inside	0.10226
	outside	0.1143

CHARACTERISTICS OF LAYERS			
Thickness sheets(m)	Thermal Cond. $(W/m°K)$	Density (kg/m^{3})	Specific heat $(J/kg.°K)$
1	1.40	1600	1670
6	0.90	1700	850
43	0.83	1800	1050
72	0.83	1800	1050
50	1.20	2000	921
103	1.20	2000	921
186	1.25	2300	753
21	2.50	2400	880

The results are in the following table.

Depth (m)	Temperature of layer(°C)	Temperature of Fluid (°C)
0	10	75,90
1,0	10,11	75,91
7,0	11,14	75,92
50,0	19,16	76,04
122,0	32,59	76,20
172,0	39,04	76,32
275,0	52,32	76,51

461,0	75,36	76,66
482,0	76,66	76,66



Production well of BOGACS

7 Analysis and discussion

The results of Zsori geothermal well show that , the calculated temperature is very low than the field temperature we measured. The explanation of that situation may be the high thickness of sand layers and clay layers, moreover, because of the high porosity and the high permeability of the sand layers, in the geothermal well, a big part of the heat transfer is convection and not conduction as we supposed.
About the results of Bogács geothermal well, there is a difference between the calculated temperature and the field temperature, because in fact the heat transfer is the combination of convection and conduction
The results show, how the cooling of the geothermal water is going on from the bottomhole to the wellhead of the wells.
Through the results, we remarked that, some hours after the production of hot water in the wells, the temperature on the wellhead depends of the flow.
For high flow, we remarked that the water cooling is very low. To do a way with the hydrodynamic perturbations between the wells and layers, it is advised to put in the annulus insulating mortar. This paper helps to know exactly the theoretical hot water temperature and permits to disposition to reduce heat losses.

8 Conclusion

In this paper, we have succeeded in presenting a sample mathematical calculation of the temperature of produced geothermal hot water, by refining the so-called Ramey-theory.
With this method, we arrived to calculate the temperature in the lower limit and on the upper limit of each layer from the bottomhole to the wellhead of geothermal hot water producing wells.
This paper gives opportunity to do a way with the hydrodynamic perturbations between the wells and layers, by using in the annulus, insulating mortar. This paper helps to know exactly the theoretical hot water temperature and permits to disposition to reduce heat losses.

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